Sayı (Number): 1

SU VAKFI

Flood and Risk Calculation

Taşkın ve Risk Hesaplaması

Zekâi Şen

Aralık (December) 2016
Istanbul - Turkey

TAŞKIN KÜLLİYESİ
TURKISH WATER FOUNDATION
FLOOD FACULTY

TAŞKIN BÜLTENİ : SAYI 1

Flood and Risk Calculation

Taşkın ve Risk Hesaplaması

Zekâi ŞEN

Basıma Hazırlayan :
Muhiddin YENİGÜN

SU VAKFI

SU VAKFI

Libadiye Cad. Doğanay Sokak No:6 Kat:4 Üsküdar İstanbul
Tel: (216) 412 3383 - Faks: (216) 412 3390
suvakfi@suvakfi.org.tr - www.suvakfi.org.tr

Flood and Risk Calculation

Zekâi Şen

Su Vakfı, Libadiye Caddesi, Doğanay Sokak No: 6, Kat 4, Üsküdar, Istanbul, Turkey

Abstract

Floods are among the most dangerous natural phenomenon that cause property and human life losses. They cannot be avoided completely, and therefore, their calculations can be achieved under a certain risk level that provides calculation methodology throughout the probabilistic, statistical and stochastic methodologies. This article provides flood risk calculation methodologies for independent and even dependent flood occurrences with consideration of the first order Markov process. The necessary tables and graphs are provided for practical uses.

1. Introduction

Floods are among the most catastrophic hydrological phenomena. They are rare but may result in the production of irregular and extreme events, and present a potential threat to both lives and property. Man-made structures such as building dams, bridges, culverts, wells roads and highways along and across wadi courses are highly in risk from floods. Flash floods forms rapidly and they flow down over extremely dry or nearly dry watercourses involving intense rainfall of short duration. Such floods are rare and variable in magnitude, time, and space, and thus flood prediction by methods that rely on extensive rainfall-runoff data are rarely proven satisfactory. In addition, the difficulty of data collection, owing to the low population density and inaccessibility of these areas, limits the reliability of the data recorded for such environments. Extreme climatic variations from year to year, and variation in annual rainfall, increase the problems of constructing probabilistic rainfall-runoff models. As a result, flood prediction for wadi channels is plagued with both risks and uncertainties, and requires complex techniques for successful achievement (El-Hames and Richards, 2000). In most zones, the runoff is produced due to convective storms which occur under unstable synoptic conditions and characterized by their high localization and intensity, and their brief duration.

The main purpose of this article is to provide flood risk formulations for independent and especially for dependent processes with small sample sizes.

2. Flood risk calculation

In the risk assessment of any engineering project it is necessary to decide first on the frequency of design flood occurrence, i.e., the return period, T, after which it is then possible to determine the magnitude of the design flood on the basis of the most suitable PDF of the hydrological variable concerned. In hydrology, the return period is defined as the average length of time over which Q will be equaled or exceeded once and only once. The random variable (r.v.) T_r which specifies the time between any two successive exceedances of the design flood is also referred to as the waiting time. Its distribution in the case of independent observations is given by Feller (1967) as

$$P(T_r \geq j) = q^{j-1} \qquad (1)$$

or

$$P(T_r = j) = pq^{j-1} \qquad (2)$$

Hence, the return period which is the expected value of waiting time is obtained as

$$T = \sum_{j=1}^{\infty} jP(T_r = j) = p\sum_{j=1}^{\infty} jq^{j-1} = \frac{1}{p} \qquad (3)$$

where p = P(x > Q), i.e., the probability of exceedence of design flood. Thus, there exists an inverse relationship between the return period and probability of exceedence.

The theoretical distribution of the return period has been illustrated in Tables 1-2 by Linsley, et.al (1958) without any analytical expression. An error has been detected in Table 1-2 in regard to the actual return period T_r which has been assigned the value of zero. However, it has been proven by Gumbel (1958) that the return period cannot be less than one. The theoretical distribution of the return period has been given by Eq. (1) the solution of which has been presented in Table 1. This table is the corrected form of Table 11-2 of Linsley (1958).

It can be seen from this table that over a long period of years 25 percent of the intervals between floods equal to or greater than the 100 year is less than about 30 years while an equal number will be in excess of about 139 years. In other words, for 75 percent safety, the capacity of a structure will not be exceeded by a flood within the next 30 years it must be designed for the 100-year flood.

Table 1. Independent process returns period theoretical distribution.

Average return period	Actual return period T_r exceeded various percentages of time. $P(T_r \geq j)$						
T	0.01	0.05	0.25	0.50	0.75	0.95	0.99
2	7.64	5.32	3.00	2.00	1.41	1.07	1.01
5	21.64	14.42	7.21	4.10	2.28	1.23	1.04
10	14.71	28.43	14.16	7.58	3.73	1.48	1.09
30	136.84	89.36	41.89	21.44	9.48	2.51	1.29
100	459.21	299.07	138.93	69.97	29.62	6.10	2.00
1000	4603.86	2995.23	1386.60	692.80	288.53	52.53	11.11

The safety and risk of a hydraulic structure which must withstand floods can be obtained in terms of return respectively as follows.

$$S=\left(1-\frac{1}{T}\right)^n \tag{4}$$

and the risk

$$R=1-\left(1-\frac{1}{T}\right)^n \tag{5}$$

The necessary tables and graphs for the application of Eq. (5) to engineering structures have been provided by Gupta (1973).

On the other hand, in the case of linearly dependent floods Eqs. (4) and (5) are not valid. The theoretical distribution of the return period can be obtained as

$$P(T_r \geq j)=q\left[1-\frac{p}{q}(1-r)\right]^{j-2} \tag{6}$$

or

$$P(T_r = j)=P(T_r \geq j)-P(T_r \geq j+1)=p(1-r)\left[1-\frac{p}{q}(1-r)\right]^{j-2} \tag{7}$$

These two last expressions reduce to the independent case conditions ρ = 0 therefore r = ρ are considered. Thus, the return period which is the expected value (average) of the r.v., T_r, can be derived after making use of Eqs. (6) and (7) leading to

$$T=\frac{q^2}{\left[1-\frac{p}{q}(1-r)\right]p(1-r)} \tag{8}$$

which proves that the return period in the case of dependent variables is not a function of probability of exceedence, p, only but also of r which is explicitly related to ρ. The theoretical distribution of the return period has been presented in Table 2 for ρ = 0.2.

Table 2 Return periods

Average return Period. T	Actual return period T_r exceeded various percentages of time. $P(T_r \geq j)$								
	0.01	0.05	0.25	0.50	0.75	0.95	0.99	p	r
2	8.83	6.02	3.21	2.00	1.29	0.88	0.80	0.500	0.5640
5	24.14	16.00	7.88	4.37	2.32	1.13	0.92	0.200	0.2818
10	48.39	31.80	15.20	8.06	3.88	1.44	1.00	0.100	0.1681
30	143.95	93.97	43.99	22.47	9.88	2.54	1.26	0.033	0.0810
100	471.22	306.88	142.53	71.75	30.35	6.21	2.00	0.010	0.0352
1000	4629.65	3012.00	1394.36	697.68	290.14	52.54	11.09	0.001	0.0065

A comparison of this table with that of Table 1 reveals the fact that the actual return periods in dependent variables are generally greater than the independent case. As a conclusion an increase in the autocorrelation coefficient causes the return period to increase. On the other hand, one can write the return period of dependent process given in Eq. (8) as,

$$S = q\left[\frac{q^2}{Tp(1-r)}\right]^{n-1} \quad (9)$$

and

$$R = 1 - q\left[\frac{q^2}{Tp(1-r)}\right]^{n-1} \quad (10)$$

respectively. The solution of Eqs. (9) and (10) can be achieved for a given pairs of p and ρ. The final numerical solutions are presented in Table 3. For any desired set of parameters and their shows that as ρ increases so does the percentage of safety at small return periods whereas for large return periods they virtually become the same.

Table 3. Markov process (ρ = 0.2) risk and safety values

p	q	r	T_r	Economic life of the project. n. in years									
				n = 10		n = 20		n = 30		n = 50		n = 100	
				Safety	Risk	Safety	Risk	Safety	Risk	Safety	Risk	Safety	Risk
0.33	0.67	0.414	3.26	0.031	0.969	0.001	0.999	0.000	1.000	0.000	1.000	0.000	1.000
0.25	0.75	0.334	4.34	0.078	0.992	0.006	0.994	0.000	1.000	0.000	1.000	0.000	1.000
0.20	0.80	0.282	5.43	0.134	0.866	0.018	0.982	0.003	0.997	0.000	1.000	0.000	1.000
0.10	0.90	0.168	10.72	0.375	0.625	0.158	0.842	0.060	0.940	0.008	0.992	0.000	1.000
0.05	0.95	0.104	21.14	0.615	0.385	0.400	0.600	0.234	0.766	0.089	0.911	0.008	0.992
0.03	0.97	0.083	35.22	0.772	0.228	0.561	0.439	0.434	0.566	0.237	0.763	0.056	0.944
0.01	0.99	0.035	102.5	0.906	0.094	0.822	0.278	0.745	0.255	0.613	0.687	0.375	0.625

This is an expected conclusion since as the time interval between two successive occurrences of an event increases the linear dependence between the events decreases and after some time period they practically become independent.

3. Small sample case

As stated previously the information content of a sample is restricted with its length; the shorter the sample the lesser is the information content. Thus, the uncertainties and risks which are always present in the hydrological data tend to become more accentuated in designs based on short samples. Even with the most powerful mathematical, statistical, probabilistic or stochastic methods, it is not possible to get rid of these risks entirely. Let us consider a sequence of n independent observations. Now, what is required is that the safety of such a sequence, i.e., the probability of nonoccurrence of design value over the sample length. If the sample length is considered to be equal to the expected life of the reservoir then the expressions derived previously become applicable. Thus, Eq. (9) is valid for safety and its solution is given in Table 4 The inspection of this table shows that there is one percent change that the average return period of the maximum event occurring in 10-year record is as low as 2.71.

Table 4. Independent process average return periods.

Number of Record. n	Safety. S				
	0.01	0.25	0.50	0.75	0.99
2	1.11	2.00	3.41	7.46	200.00
5	1.66	4.13	7.73	17.88	498.00
10	2.71	7.73	14.93	35.26	995.50
20	4.86	14.93	29.36	70.00	1990.50
60	13.50	43.78	87.06	209.00	5970.45

3.1 Independent process

If the successive observations are independent then there are two simple and complementary events and their probabilities, namely, the probability of occurrence,

$p = P(x > Q)$ and probability of nonoccurrence, $q = 1 - p = P(x \leq Q)$ from which it is possible to evaluate various probabilities of any compound event of the phenomenon considered. For instance, S given is a compound event and in terms of the simple event probabilities it becomes

$$S = q^n \quad (11)$$

The corresponding risk can be found as,

$$R = 1 - q^n \quad (12)$$

This expression yields a straight line on a semi logarithmic graph paper, which is very useful in practical applications of water resources systems design in calculating the probability of nonoccurrence of design variate, Q, given the level of safety and the expected life of the project. For instance, if the planner is interested in designing his project for n=10 years with 90% safety, that is to say, in the long run the project will not fail 90% of the time, then from Figure 1, q=0.99 is obtained, hence the probability of occurrence is p=0.01. The planner may be able to find the magnitude of design variate by adopting a suitable PDF to data at hand with p=0.01. For example, if the adopted PDF is normal then the standard design variate corresponding to p=0.01 is $x \cong 2.32$ whereas the actual design variate can be found as

$$Q = \mu + x\sigma \quad (13)$$

where μ and σ are the mean and standard deviation of the normal PDF. Thus, the resulting design variate is safe at 99% level.

On the other hand, if the expected life of the project is to be ignored, say, to 20 years with q=0.99 then the safety can be found as 80%. Hence, as the expected project life increases by safety of the design decreases. This is logical, since as the project life increases, the probability for exceedence of design variate also increases.

Sometimes, it is desirable to know the safety of an already existing engineering structure. For instance, if the structure has been designed originally for an expected life of n=30 years with q=0.99, after its completion the calculated safety will be 73.9%. If the design value has not been exceeded for the first, say 20 years, then the safety of the same structure for the remaining 10 years will be 90.0% which is considerably greater than the original safety. An important conclusion is that as the number of years without any increases the safety of structure will improve reducing the risk.

4. Risk calculations in water sciences

The simple safety, S, can be defined as the probability of nonoccurrence of the hydrologic variable, x, to be greater than the design magnitude, Q, over the system's economic life, n. If the sequence of future likely occurrence of x is $x_1, x_2, \ldots, x_n$ then the joint probability of nonoccurrence is defined as

$$S = P(x \leq Q) = P(x_1 \leq Q, x_2 \leq Q, .., x_n \leq Q) \quad (14)$$

On the other hand, the simple risk, R, is as a complementary event to safety and hence,

$$R = P(x > Q) = 1 - S \quad (15)$$

The calculation of the multivariate probability term on the right hand side of Eq.(1) is dependent on the structure of the variate considered and, in general, it can be calculated by multiple integration of the multivariate probability distribution function, (PDF) through tetrachoric series expansion (Saldarriaga and Yevjevich, 1970). However, in the case of simple dependence structure such as the first order Markov dependence factorizes into various terms which are explained by Şen (1976).

4.1 First order Markov process :

The simplest form of serial dependence in hydrologic process is modeled by first order Markov process which is expressed as

$$X_i - \mu = \rho(X_{i-1} - \mu) + \sigma\sqrt{1-\rho}\varepsilon_i \quad (16)$$

where X_i's are random variables, ρ is the first order autocorrelation coefficient and ε_i is a white noise term with zero mean and unit variance. Simple risk and safety can be expressed in terms of probabilistic descriptions of the phenomenon by considering project life and design magnitude. The complete probabilistic description of a first order Markov process truncated at a specified standard design magnitude Q can be achieved through a Markov chain with two states only corresponding to the events of exceedence ($X_i>Q$) and non-exceedence ($X_i\leq Q$). The whole probabilistic description of such a process can be obtained in terms of two states $p = P(X_i>Q)$ and $q = 1-p = P(X_i\leq Q)$ and four successive transition states with conditional probabilities $P(X_i>Q|X_{i-1}>Q)$, $P(X_i>Q|X_{i-1}\leq Q)$, $P(X_i\leq Q|X_{i-1}>Q)$, and $P(X_i\leq Q|X_{i-1}\leq Q)$ for i=2,3,........,n. These transitional state probabilities are, in fact, the transitional probabilities from one state to the next at two successive time instances. It is to be noticed that these conditional probabilities should satisfy the following equalities which are

$P(X_i>Q|X_{i-1}>Q)+P(X_i\leq Q|X_{i-1}>Q)$,

and

$P(X_i>Q|X_{i-1}\leq Q),+P(X_i\leq Q|X_{i-1}\leq Q)$

Hence, there are three probability values, namely, p, $P(X_i>Q|X_{i-1}>Q)$ and $P(X_i\leq Q|X_{i-1}\leq Q)$ which are needed in the complete probabilistic description of the process concerned. These probabilities can be estimated from a given realization of the process as an observed time series provided that additionally the design magnitude is also specified. The first order autocorrelation coefficient, ρ in terms of the aforementioned probabilities become (Şen, 1991) as

$$\rho = [P(X_i>Q|X_{i-1}>Q)-P(X_i\leq Q|X_{i-1}\leq Q)]+ [P(X_i\leq Q|X_{i-1}>Q)-P(X_i>Q|X_{i-1}\leq Q)]$$

On the other hand, Şen (1976) defined $P(X_i>Q|X_{i-1}\leq Q)$ as the first order autorun coefficient, r. Hence, the explicit forms of the conditional probabilities become in terms of readily estimable basic questions from a given time series as

$$P(x_i>Q|x_{i-1}>Q) = r \quad (17)$$

$$P(x_i>Q|x_{i-1}\leq Q) = (p/q)(1-r) \quad (18)$$

$$P(x_i\leq Q|x_{i-1}\leq Q) = 1-(p/q)(1-r) \quad (19)$$

and

$$P(x_i\leq Q|x_{i-1}>Q)= (1-r) \quad (20)$$

where $P(x_i\leq Q|x_{i-1}\leq Q)$ is given in an integral form by Cramer and Leadbetter (1967) as

$$P\left(x_i \leq Q|x_{i-1} \leq Q\right)=q+\frac{1}{2\pi q}\int_0^{\rho} e^{-z^2/2(1+z)}\left(1-z^2\right)^{1/2}dz \quad (21)$$

The numerical solution of this expression for various ρ and Q values have been given in by Şen (1976). Especially for the first order Markov process factorizes into the following parts

$$S=P(x_i \leq Q)\prod_{i=2}^{n} P\left(x_i \leq Q|x_{i-1} \leq Q\right) \quad (22)$$

or

$$S =q\left[1-\frac{p}{q}(1-r)\right]^{n-1} \quad (23)$$

which for ρ = 0.0 i.e., r = p reduces to $S = q^n$ which has been already given by Yeh (1970) for independent processes. The risk can be found as,

$$R =1-q\left[1-\frac{p}{q}(1-r)\right]^{n-1} \quad (24)$$

which again for independent process case reduces to $R = 1-q^n$

At this stage a question might be asked as to how does the correlation effect the simple risk? For instance, let a designer have the information that a project is going to function over n=10 years with p=0.01 occurrence probability of design variate such as flood or precipitation and that the variate has a first order Markovian structure with ρ=0.2. If the risk is calculated without with the assumption that the process is independent then R=1-(0.99)10=0.00956 dependence with ρ=0.2 and correspondingly r=0.0192, Eq. (24) yields R=0.0198. This simple correlation proves that an increase in ρ decreases risk significantly and in turn increases the safety as a result of which the size hydraulic structure and hence the cost decreases. This simple calculation indicates hoe significant is the serial correlation coefficient's effect in the assessment of risk and safety during engineering planning.

4.2 Risk by autorun function

A branch of hydrology known as "stochastic hydrology" deals with uncertainty in water resources evaluation. Prevailing hydrological conditions in an area are controlled by the distribution and occurrence of natural phenomena. Although various geometrical (extent, areal coverage, elevation, slope, etc.), structural (fault, folds, fractures, fissures, joints), textural (porosity, specific surface, grain size composition, sorting, etc.), hydraulic and geological properties are random in various degrees, hydrologists generally use deterministic approaches to problem solving. Deterministic approaches give satisfactory single point results at the local scale, but on a regional scale (because of the heterogeneities, discontinuities, and anisotropies) use of such a point value fails to provide a reliable solution. The preliminary deterministic results are not likely to be representative for other parts or larger portions of the hydrologic system. Hence, the simplest way of regionalizing a variable or a parameter is possibly through probabilistic and statistical techniques by defining risk concept.

Any measured data either in the field or in the laboratory represent quantities that are numerically different from each other. Randomness does not mean that the hydrological parameter values are spatially independent, but rather that they are not predictable at any particular site even though their values might be available at neighboring sites. For instance, the runoff coefficient for a watershed will be different at different locations. In order to obtain representative and reliable estimates for the whole watershed, the hydrologist often uses a single deterministic value such as the mean or the median or another subjective value. In fact, use of a deterministically fixed value makes it easier to use available mathematical procedures. However, such simplifications bring with the following disadvantages.

i) It is not possible to make risk assessments of the hydrological problem,
ii) The results cannot be related with the real surface or subsurface hydrology,
iii) Theoretical distribution functions cannot be established, and
iv) Correlation analysis cannot be performed between different hydrological parameters.

Uncertainties are differences between the true and estimated models or parameters. One way of decreasing uncertainty is by selecting the model that best represent the physical reality of the system. Once the model is judged as adequate, it remains to evaluate the corresponding uncertainties. Hence, two kinds of uncertainties are usually encountered in hydrological studies, namely, model and parameter uncertainties. Model uncertainty results from events that are not known, and at best the model is only a close approximation of reality. On the other hand, parameter uncertainty results because model parameters estimated from a scarce amount of data. Parameter uncertainty may be estimated by finding the distribution of parameter estimates and by using models with parameters sampled from this distribution.

The probabilistic approach is one technique used by hydrologists to characterize spatial and temporal variability, and to express a very simple criterion for goodness of estimation. Such an approach assumes independence of hydrologic events and needs at most a graphical representation of data (histograms and scatter diagrams), and hence proves to be more practical. Therefore, a simple approach has been devised in this paper on the basis of the autorun function to assess the risk associated with estimation and prediction of variables. In fact, risk assessments are invariably based on the concept of simple dependence. The autorun approach with persistence as presented in this paper is a prerequisite for more concise hydrological studies of risk.

A characteristic of risk is that it has a definable probability distribution function (PDF), which, in practice, is presented, in the form of a histogram. The methodology described in this paper has the following important implications and advantages over commonly used classical deterministic techniques:

i) A quantitative way of assessing hydrological risk by considering dependence based on the autorun function for water resources evaluation,
ii) It provides a flexible procedure to have various alternative estimates,
iii) It provides an autorun approach to determine the best estimate in the sense that it is the most likely value to occur in dependent processes, and,
iv) It provides a way of incorporating the risk concept in dependence structure of a given hydrological event.

The risk, R, of having an exceedance once after n years can be expressed probabilistically as,

$$R = 1-P(x_i<x_0, x_{i-1}<x_0, x_{i-2}<_{x0}, \ldots x_{i-n}<x_0) \quad (25)$$

which can be factored down to the following expression for simplest dependence structure, as,

$$R = 1 - P(x_i < x_0) \prod_{i=2}^{n} P(x_i \le x_0 / x_{i-1} \le x_0)$$

or consideration of exceedance, p, and non-exceedence, q, probabilities in addition to the autorun coefficient, r, it is possible to rewrite this expression successively as,

$$R = 1-q[1-(p/q)(1-r)]^{n-1} \quad (26)$$

which is the simplest expression of dependent hydrologic process risk. This final expression has been further simplified through the classical least squares curve fitting technique as

$$R = 1-q^{[1+p(n-1)/r]} \quad (27)$$

Risk-autorun coefficient relationships according to Eqs. (26) and (27) are shown in Figure 1. The two equations yield close results to each other. Since, Eq. (27) is simpler and easier to calculate, it should be preferred in practical studies. Figure 2 shows the change of risk with autorun coefficient for different exceedance probabilities and record lengths.

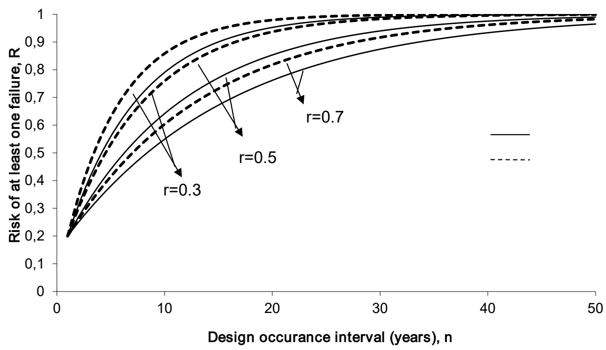

Figure 1 Risk-autorun-project life relationship

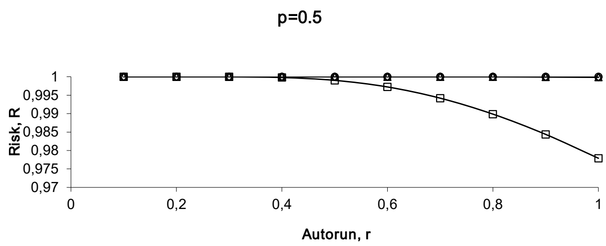

Figure 2 Risk-autorun and exceedence probabilities

In the case of independent process, r=0.5 and for p=q=0.5 this last expression leads to

$$R = 1-q^n \quad (28)$$

This expression has already been derived by Feller (1967). Finally, the return period of the dependent process was given by Şen (1999) as

$R = 1\text{-}q\{q^2/[Tp(1\text{-}r)]\}^{n-1}$ (29)

where T is the average return period. Of interest is the way that the correlation affects the simple risk. For instance, let a designer have the information that a project is going to function over n=10 years with p=0.01 occurrence probability of a design variate such as flood or precipitation and that the variate has a first order Markovian structure with ρ=0.2. If the risk is calculated with the assumption that the process is independent then from Eq. (29), $R=1-(0.99)^{10}=0.0956$; dependence with ρ=0.2, and correspondingly, r=0.0227, Eq. (24) yields R=0.0487. This simple correlation proves that an increase in ρ decreases risk, hence the size of the hydraulic structure and the cost also decrease. This simple calculation indicates how significant is the autorun coefficient's effect in the assessment of risk and safety during engineering planning. Also, the theoretical distribution of the return period T_r can be obtained from Eq. (29) as

$$P(T_r = j) = p(1-r)[1-\frac{p}{q}(1-r)]^{j-2} \quad (30)$$

By considering Eqn. (24) it is possible to simplify this expression as

$$P(T_r = j) = \frac{p}{q}(1-r)q^{[1+\frac{p(j-1)}{r}]} \quad (31)$$

Herein, P (T_r=j) means the probability of return period to be equal to j months, years or days as applicable. The graphical representation of Eq. (31) is presented in Figure 3.

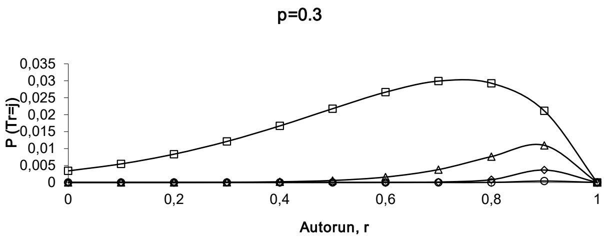

Figure 3 Return period-autorun relationship

It is possible to show the probability of return period change by autorun coefficient as in Figure 4 for different record lengths.

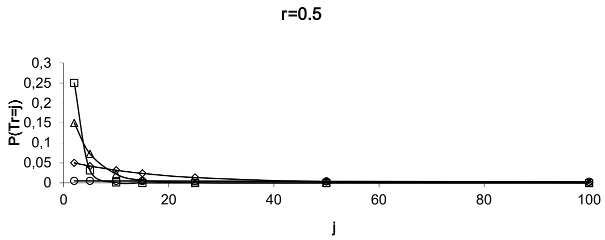

Figure 4 Return period-autorun-project life relationship

References

El-Hames, A. S. and Richards, S. K.: An integrated, physically based model for arid region flash flood prediction capable of simulating dynamic transmission loss, Hydrol. Process., 12, 1219–1233, 1998.

Feller, W., 1967. An introduction to probability theory and its application. Vol.1, John Wiley and Sons, Inc., New York, 509 pp

Gumbel, E.J., 1958. Statistics of Extremes. Colorado State University Press, New York, 375 pp.

Gupta, V.L., 1973. Information content of time-variant data. J. Hydraul. Div., ASCE, Vol. No.HY3, Proc. Paper 9615 : 383-393.

Linsley, R.K., Kohler, M.A., and Paulhus, J.L.H., 1958. Hydrology for engineers. McGraw- Hill Book Co., Inc. : 258-259.

Saldarriaga, J., and Yevjevich, V., 1970. Application of run-lengths to hydrologic series. Hydrology Paper 40, Colorado State University, Fort Collins, Colorado.

Şen, Z., 1976. Wet and dry periods of annual flow series. J. Hydraul. Div., ASCE, Vol. 102, No. HY10, Proc. Paper 12457 : 1503-1514.

Şen, Z., (1991). Probabilistic modeling of crossing in small samples and application of runs to hydrology. J. HYdrol., Vol. 124: 345-362.

Şen, Z., (1999). Simple risk calculations in dependent hydrological series. Hydrological Sciences-Joumat-des Sciences Hydrologiques, 44(6),: 871-878.

Yen, B.C., (1970). Risks in hydrologic design of engineering projects. J. HYdraul. Div., ASCE, Vol. 96(HY4), Proc. Paper 7229: 959-966.

SU VAKFI

Libadiye Cad. Doğanay Sokak No:6 Kat:4 Üsküdar İstanbul
Tel: (216) 412 3383 - Faks: (216) 412 3390
suvakfi@suvakfi.org.tr - www.suvakfi.org.tr